Another Dantzig-Wolfe Decomposition for the Vehicle Routing Problem with Capacity and Time Windows

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Branch and Price approaches are considered as the most efficient to solve Vehicle Routing Problems with Capacity and Time Windows (VRPCTW). They are based on the Branch and Bound algorithm in which the bound is computed by a column generation procedure. More precisely, the Dantzig-Wolfe decomposition is applied in order to reformulate the problem as a partitioning problem in which the variables are the different feasible routes. With such a formulation, the problem is a linear program with variables 0/1. However, if the number of constraints is polynomial, the number of variables (the feasible routes) is exponential. Due to huge number of variables, the global problem is decomposed into two levels in order to apply the column generation scheme. Firstly, we formulate a reduced, master, problem, secondly we look for the solution of the sub-problem. The routes for the master problem are examined in details in order to choose the one to which the minimal reduced cost is assigned. If this reduced cost is negative, the route is added to the subset of variables of the master problem and the process is repeated. Let us notice that the sub-problem itself is NP-difficult. The last remark is that the solution finally obtained by the column generation procedure is not always an integer solution and it can be only used as a lower bound in the Branch and Bound procedure.

We propose another decomposition of the VRPCTW. Our idea is to use variables in the form of the pair \((\text{arc}; \text{state})\) where \text{state} describes the date and the charge of the vehicle which passes through \text{arc}. With such variables, the modelisation of the problem is strongly modified. But we can show that this formulation can be also obtained by the Dantzig-Wolfe decomposition. Moreover, the optimal solution is the same that the one obtained with the classical decomposition. With this modelisation, the master problem is to find the set of variables \((\text{arc}; \text{state})\) which compose the solution of the VRPCTW. Two types of constraints have to be considered: the first ones are the classical constraints which impose that each customer is visited only once (partition constraints); the second ones impose that a vehicle does not lose any resources between the arrival and the departure at the customer (constraints of conservation of flow).

The point is that with our formulation, the number of variables of the master problem is significantly reduced comparing with the classical decomposition (the number is pseudo-polynomial instead exponential). On the other hand, the number of constraints is lighter: it becomes pseudo-polynomial while it was polynomial in the first decomposition.

The solution is initialized with a subset of variables, and also with a subset of constraints: if the partition constraints are taken under consideration, the conservation flow constraints are initially relaxed. On the first step, the variables with the best reduced cost are selected and added to the master problem. On the second step, the constraints which are unsatisfied in the current solution are added.

Intensive tests have been performed with the classical Solomon's benchmark. The comparisons with the best existing methods show that the results obtained with our approach are promising.